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Examiners' Report

Summer 2015

Pearson Edexcel GCE in Core Mathematics C4 (6666/01)

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General Introduction

This paper was a good test of Core 4 material and discriminated well across students of all abilities. There were plenty of opportunities for E-grade students to gain some marks in all 8 questions on this paper. There were also some testing questions involving vectors, differential equations and integration that allowed the paper to discriminate across the higher ability levels.

It was noticeable on this paper that some students did not read a number of the questions properly. In Q3(c), a number of students tried to find the area of R by applying $\int_0^{4\ln 2} x\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x$ instead of applying $\int_0^{4\ln 2} (4x - x\mathrm{e}^{\frac{1}{2}x})\,\mathrm{d}x$. In Q6, a significant minority of students having solved their equations simultaneously to find $\lambda = 4$ and $\mu = -1$, proceeded to find a correct $p = 15$ in part (b), but forgot to find the coordinates of A in part (a). In Q7(c), a large number of students did not realise that the population was measured in thousands and so substituted $P = 4000$ and not $P = 4$ into the given expression. Therefore examiners suggest that teachers could encourage their students to read questions carefully, to underline key words or ideas and to take the time to understand what the question is asking – not what they assume it is asking.

The standard of algebra seen by examiners was generally good, although a number of students made basic sign, bracketing or manipulation errors in Q1(a), Q2, Q3(c), Q4(a), Q4(b), Q4(c), Q5, Q6, Q7(c) and Q8. In summary, Q1, Q2, Q3(a), Q3(b), Q4(a), Q4(b), Q4(c), Q5 and Q7(a) were a good source of marks for the average student, mainly testing standard ideas and techniques; and Q3(c), Q4(d), Q6(a), Q7(b) and Q8(a) were discriminating at the higher grades. Q6(b), Q7(c) and Q8(b) proved to be the most challenging questions on the paper.

Report on individual questions

Question 1

This question was well answered with about 41% of students gaining full marks and about 77% gaining at least 6 of the 8 marks available.

In part (a), most students started by manipulating $(4+5x)^{\frac{1}{2}}$ to give $2\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$, although a few incorrectly wrote $4\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$ or $\frac{1}{2}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$ The majority were able to use a correct method for expanding a binomial expression of the form $(1+ax)^n$. A variety of incorrect values of a such as 5, $-\frac{5}{4}$ or $\frac{5}{2}$ were seen at this stage. Some students, who expanded $\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$ to give $1+\frac{5}{8}x-\frac{25}{128}x^2+\ldots$, then forgot to multiply their expansion by 2 to give the answer to part (a). Sign errors, bracketing errors, not dividing their third term in their expansion by 2! and simplification errors were also seen this part.

In part (b), the majority of students substituted $x=\frac{1}{10}$ into $(4+5x)^{\frac{1}{2}}$ and obtained $\frac{3}{2}\sqrt{2}$, but a minority who did not give their answer in the form $k\sqrt{2}$ gave an answer of $\frac{3}{\sqrt{2}}$.

In part (c), most students realised that they had to evaluate their answer to part (a) with $x=\frac{1}{10}$. About half of them, however, failed to recognise the implication of part (b), in that this evaluation needed to be multiplied by $\frac{2}{3}$ in order to find an approximation for $\sqrt{2}$.

Question 2

Students generally performed well on this question with about 39% gaining full marks and about 65% gaining at least 9 of the 11 marks available.

In part (a), many students were able to differentiate correctly, factorise out $\frac{dy}{dx}$, and rearrange their equation to arrive at a correct expression for the gradient function. A minority did not apply the product rule to differentiate $-3xy$, and a small number left the constant term of 64 on the left hand side of their differentiated equation. The most common errors seen in part (a) were sign errors, usually in differentiating $-3xy$, but also, occasionally, when manipulating their terms to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject.

In part (b), most set the numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero, whilst a few set $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ in their '$2x - 3y - 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 8y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$'. The majority of students achieved either $y = \frac{2}{3}x$ or $x = \frac{3}{2}y$ and substituted this result into $x^2 - 3xy - 4y^2 + 64 = 0$ to achieve an equation in one variable. Manipulation and bracketing errors led some students to give $x^2 = A$ or $y^2 = A$, where A was negative. Whilst a majority of students reached a correct $x^2 = \frac{576}{25}$ or $y^2 = \frac{256}{25}$, a significant minority failed to realise that these equations have two solutions. Some students, having obtained their values of x (or y), then proceeded to substitute these into $x^2 - 3xy - 4y^2 + 64 = 0$, rather than using the much simpler $y = \frac{2}{3}x$ or $x = \frac{3}{2}y$. Few students having found a correct $x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$, incorrectly stated their coordinates as either $\left(\frac{16}{5}, \frac{24}{5}\right)$ or $\left(\frac{24}{5}, -\frac{16}{5}\right)$.

Question 3

This question was well answered with about 20% of students gaining full marks and about 48% gaining at least 7 of the 8 marks available.

In part (a), the majority of students set $y = 0$ and either factorised out x or divided both sides of their equation by x in order to obtain $e^{\frac{1}{2}x} = 4$. Although some gave their answer as $x = 2\ln 4$ or $x = \ln 16$ which were not in the required form, most correctly found $x = 4\ln 2$.

In part (b), most students recognised the need to use integration by parts and many fully correct solutions were seen. A few students labelled u and $\frac{\mathrm{d}v}{\mathrm{d}x}$ the wrong way round and a common error was to integrate $e^{\frac{1}{2}x}$ to give either $\frac{1}{2}e^{\frac{1}{2}x}$ or even $\frac{2}{3}e^{\frac{3}{2}x}$. During the second stage of the method few students integrated $2e^{\frac{1}{2}x}$ to give $e^{\frac{1}{2}x}$. Some students omitted "$+c$" from their final answer but ***on this occasion*** it was condoned as the purpose of the whole question was to find the area of the shaded region R.

A minority of students who did not read part (c) carefully provided a solution to find $\int_0^{4\ln 2} xe^{\frac{1}{2}x}\,dx$ instead of $\int_0^{4\ln 2} (4x - xe^{\frac{1}{2}x})\,dx$. Some students made sign errors by simplifying $x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)$ incorrectly to give $x^2 - 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$. A minority did not bother to substitute 0 into their integrated expression, but incorrectly assumed the result of this would be 0. Only a minority of students found the correct exact area of $32(\ln 2)^2 - 32\ln 2 + 12$. Some of these students tried to simplify $32(\ln 2)^2 - 32\ln 2 + 12$ further to give either $64\ln 2 - 32\ln 2 + 12 = 32\ln 2 + 12$ or $8(\ln 2)^2 - 8\ln 2 + 3$ (dividing each term by 4) and so lost the final accuracy mark.

Question 4

This question was well answered with the majority of students scoring full marks in the first three parts. Part (d) offered good discrimination for the more able students. About 27% of students gained full marks and about 66% gained at least 8 of the 11 marks available.

In part (a), most students equated the **i** components of l_1 and l_2 to find μ and substituted μ into l_2 to find the coordinates of point *A*. A significant number, however, took the longer route by equating the **i** and **j** components of l_1 and l_2 and solving simultaneously to find the values of λ and μ with some finding the value of *p* first. Surprisingly, some students did not substitute their value for μ into l_2, and consequently lost both marks in part (a).

In part (b), most students found *p* by substituting their values of λ and μ into an equation formed by equating the equating the **k** components of l_1 and l_2. Other students found *p* by substituting their value of λ into $p - 3\lambda$ = their **k** component of *A*. Most mistakes in parts (a) and (b) arose from sign errors, algebraic slips or from substituting an incorrect value for λ or μ into their equations.

In part (c), a large majority of students were able to find the correct acute angle by taking the dot product between the direction vectors of l_1 and l_2. The majority worked in degrees with only a few answers given in radians. Some students applied the dot product formula between multiples of the direction vectors (using their λ and μ from parts (a) and (b)), which usually led to an obtuse angle; however, most realised that they needed to subtract this angle from 180° in order to find the correct acute angle. A number of students used incorrect vectors such as either $5\mathbf{i} - 3\mathbf{j} + 15\mathbf{k}$, $8\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ or $5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ in their dot product equation. Other mistakes included sign and computational errors in applying the dot product formula. A few lost the final accuracy mark by writing their angle correct to one decimal place instead of the required 2 decimal places.

Only a minority of students were able to successfully answer part (d). Those who drew a simple diagram to represent the situation were able to find the length AB, and complete the question by using elementary trigonometry, although some used the tangent or cosine of their part (c) angle rather than the sine. The majority of students, however, either just left part (d) blank or thought that AB or OB was the length they were looking for. Some tried more complicated methods such as finding a general expression for $\overrightarrow{BX}$ and taking the scalar product of this vector with the direction of l_1 and equating to zero. The working for this method is more complicated $\left(\lambda = \frac{39}{5}\right)$ but a significant number of correct solutions of this type were seen.

Question 5

Students generally performed well on this question with about 43% gaining full marks and about 57% gaining at least 5 of the 6 marks available.

In part (a), a significant number of students differentiated $y = 4t + 8 + \dfrac{5}{2t}$ incorrectly to give $\dfrac{dy}{dt}$ as either $4 - 5(2t)^{-2}$, $4 - 10t^{-2}$ or $-\dfrac{5}{2t^2}$. The majority of students were able to apply the process of parametric differentiation although a small number did not substitute $t = 2$ into their expression for $\dfrac{dy}{dx}$. Few students attempted to answer part (a) by differentiating their Cartesian equation of C.

In part (b), most students were able to eliminate t in order to form an equation in only x and y. Although many correct solutions were seen, algebraic and arithmetic errors were seen in the work of a significant number of students. For example, some students simplified the algebraic fraction $\dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$ to give $\dfrac{20}{x-3}$, whilst others simplified $y = x - 3 + 8 + \dfrac{10}{x-3}$ to give $y = x - 5 + \dfrac{10}{x-3}$. A minority of students used incorrect methods to rationalise the denominator with some simplifying a correct $y = x + 5 + \dfrac{10}{x-3}$ to give either $y = \dfrac{x(x-3) + 5 + 10}{x-3}$ or $y = \dfrac{(x+5)(x-3) + 10(x-3)}{x-3}$.

Question 6

This question discriminated well across students of all abilities, with about 20% gaining full marks and about 53% gaining at least 5 of the 8 marks available.

The majority of students started part (a) by differentiating $x = 1 + 2\sin\theta$ to give $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ and usually proceeded to use this in their substitution. Some students, however, failed to substitute for the $\mathrm{d}x$ part of the integral with some replacing '$\mathrm{d}x$' with '$\mathrm{d}\theta$'. A surprising number of students substituted $x = 1 + 2\sin\theta$ into $\sqrt{(3-x)(x+1)}$ to obtain $\sqrt{(2+2\sin\theta)^2}$ by making sign or bracketing errors. Most of those who correctly obtained $\sqrt{(4-4\sin^2\theta)}$, usually recognised the need to use the trigonometric identity $1-\sin^2\theta = \cos^2\theta.$ and proceeded to give a correct proof. Some students lost the final mark in part (a) by not providing sufficient evidence to show the change in limits.

In part (b), the majority of students who attempted to integrate $k\cos^2\theta$ realised the need for using $\cos 2\theta \equiv 2\cos^2\theta - 1$. Whilst the double angle formula was generally quoted correctly, this did not always lead to a correct expression for integration as a result of sign or bracketing errors. The majority of students, because of either incorrect integration, not using the correct value of k or struggling to apply the limits of $-\frac{\pi}{6}$ and $\frac{\pi}{2}$ were not able to find the correct answer of $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$. Those students who gained 0 marks in part (b) usually integrated $\cos^2\theta$ to give expressions such as $\frac{\cos^3\theta}{3}$ or $\frac{\cos^3\theta}{-3\sin\theta}$.

Question 7

This question discriminated well across students of all abilities, with about 14% gaining full marks and about 49% gaining at least 9 of the 13 marks available. It was common for some students to score all 3 marks in part (a), only the first 4 marks in part (b) and 1 or 2 marks in part (c).

In part (a), the majority of students split up $\frac{2}{P(P-2)}$ as $\frac{A}{P} + \frac{B}{(P-2)}$ and wrote down the correct identity $2 \equiv A(P-2) + BP.$ Many found the correct values of A and B by either substituting $P = 2$ and $P = 0$ into their identity or by equating coefficients. Occasionally there were arithmetic slips or a failure to write out the partial fractions when the constants A and B had been found.

In part (b), most students were able to separate the variables, although some did this incorrectly usually by putting the "2" in the wrong position. Most made the link with part (a) and integrated to give a correct $\ln(P-2)-\ln P = \frac{1}{2}\sin 2t + c$. Common errors at this stage included integrating $\cos 2t$ to give $-\frac{1}{2}\sin 2t$ or integrating $\frac{2}{P(P-2)}$ to give $2\ln(P(P-2))$ or writing $\ln(P-2)-\ln P = \frac{1}{4}\sin 2t + c$. Some students omitted a constant of integration from their working whilst others wrote "$+ c$" in their working but did not attempt to use the boundary conditions $t = 0$, $P = 3$ to find the value of c. Most students were able to apply the subtraction (or sometimes the addition) law of logarithms correctly for their expression but a significant number of students struggled to correctly remove the logarithms from their integrated equation, with the incorrect manipulation of $\ln\left(\frac{P-2}{P}\right) = \frac{1}{2}\sin 2t + \ln\left(\frac{1}{3}\right)$ leading to $\frac{P-2}{P} = e^{\frac{1}{2}\sin 2t} + \frac{1}{3}$ usually seen. Those who eliminated their logarithms successfully usually provided a full method to make P the subject of their equation, although it must be noted that students should be encouraged to show all steps in their working, especially when they are required to prove a given answer.

Only a minority of students gained all 3 marks in part (c). The most common error was to substitute $P = 4000$ into the given expression. Only few realised their error when they were not able to solve the resulting $\sin 2t = 2.19...$, and so recovered to use $P = 4$. A significant number of students evaluated $t = \frac{1}{2}\arcsin(2\ln 1.5)$ in degrees rather than in radians. It was also disappointing to see students at this level who manipulated $\sin 2t = 2\ln 1.5$ to give $t = \frac{2\ln 1.5}{\sin 2}$.

Question 8

This question discriminated well across the higher ability students, with about 10% gaining full marks and about 38% gaining at least 6 of the 10 marks available.

In part (a), a majority of students differentiated $y = 3^x$ to give $\frac{dy}{dx} = 3^x \ln 3$ with some deriving this result from first principles. A minority writing 3^x as $e^{x\ln 3}$, correctly found $\frac{dy}{dx} = (\ln 3)e^{x\ln 3}$. A relatively small number of students differentiated incorrectly to give $\frac{dy}{dx}$ as either 3^{x-1}, $x3^{x-1}$ or 3^x. Some students did not find a numerical value of the gradient function at $(2,3)$ and tried to solve the equation $-9 = 3^x \ln 3(x-2)$. Many, however, used their gradient function to find a linear equation for the tangent to the curve at $(2, 3)$, put $y = 0$ and usually found a correct

$x_Q = 2 - \frac{1}{\ln 3}$ or $\frac{18\ln 3 - 9}{9\ln 3}$ or $\log_3(9\mathrm{e}^{-1})$. Some students, however, simplified a correct $-9 = 9\ln 3(x-2)$ to give an incorrect $-9 = 9\ln(3x) - 18\ln 3$, which they proceeded to solve; whilst others made sign errors and found $x = 2 + \frac{1}{\ln 3}$.

In part (b), the majority of students were able to apply volume formula $\pi\int y^2\,\mathrm{d}x$ correctly to give $\pi\int (3^x)^2\,\mathrm{d}x$ or $\pi\int 3^{2x}\,\mathrm{d}x$ or $\pi\int 9^x\,\mathrm{d}x$, although a number of students used incorrect formulae such as $2\pi\int y^2\mathrm{d}x$ or $\int y^2\mathrm{d}x$ or even $\int y\,\mathrm{d}x$. Some students, however, incorrectly simplified $\pi\int (3^x)^2\,\mathrm{d}x$ to give $\pi\int 3^{x^2}\,\mathrm{d}x$ or $\pi\int 9^{2x}\,\mathrm{d}x$. Many students integrated 3^{2x} incorrectly to give expressions such as $\frac{3^{2x}}{\ln 3}$, $3^{2x}(\ln 3)$, $3^{2x}(2\ln 3)$, $\frac{3^{2x+1}}{2x+1}$ or $\frac{3^{2x}}{2}$. Although the majority applied the correct limits of 2 and 0 to their integrated expression, a considerable number applied limits of $2 - \frac{1}{\ln 3}$ and 0. A minority did not substitute the lower limit of 0 into their integrated expression, but incorrectly assumed the result of this would be 0, whilst others assuming $a^0 = a$, applied 0 to $\frac{3^{2x}}{2\ln 3}$ and found $\frac{3}{2\ln 3}$. The majority of students used the hint and applied $V = \frac{1}{3}\pi r^2 h$ to find the volume of the cone, although a small number struggled to find this volume by applying $\pi\int_{2-\frac{1}{\ln 3}}^{2}\left(9x\ln 3 - 18\ln 3 + 9\right)^2\mathrm{d}x$. Common errors for finding the volume of the cone included mixing up the values for r and h, using h = their x_Q rather than $h = 2 -$ their x_Q, or making sign errors when applying $h = 2 - \left(2 - \frac{1}{\ln 3}\right)$. Whilst a significant minority of students applied a full method of $\pi\int_0^2 3^{2x}\,\mathrm{d}x - V_{\text{cone}}$, only about 10% of the candidature obtained the correct answer of $\frac{13\pi}{\ln 3}$.

Grade Boundaries

Grade boundaries for this, and all other papers, can be found on the website on this link:
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